Litvin D., IAN-105a group

Surprising and wonderful behavior of an Iteration Sequence

Iteration sequence of $z_{i+1} = z_{i+1}^2 + C$, $i = 1,2,...$ for complex numbers C and an initial value z_0 are famous as an example of a so called deterministic chaos appearance that leads to Julia fractal [1]. Indeed, if we take various C and z_0, the sequence z_i, $i = 1,2,3,4,...$ being plotted on the surface $x = re(z_i)$, $y = im(z_i)$ presents quite different pictures. For some choose of complex z_0 and C, we get pictures that may be called a "Galaxy" with several beams rotated in either clockwise or counterclockwise direction; in the case $C = -.3905 - .5868i$ and $z_0 = 0$ we get a leaf, etc [2]. Of course, both roots of the square equation $z^2 - z + C$ are to be plotted as well, and the initial value z_0 is to be not far from them.

The aim of my work was to consider several such "Galaxies" and to look if there is any order, any law in the issuing consequent iteration sequence points. It was an expectation that all these points follow with a certain periodicity. To check this hypothesis, it was foreseen in the program to change the color of dots to be plotted with a certain periodicity. For example, for periodicity $T = 3$ the following MATLAB-code may be used:

```
z= ….; C= …; % Choosing iteration sequence parameters
for i=1:inf   % unlimited Iteration Number  first 500/4=125 points are of Color1
   ColorNumber=mod(i,3); % Modulus after division of integer i to 3, i.e. either 0,1 or 2
   if ColorNumber==0
      Color='blue';  % Choosing color in each of 3 cases
   elseif  ColorNumber==1
      Color='red';      % Choosing RED color in this particular case
   else
      Color='green';
   end
     z=z^2+C;
   h=plot(z, '.', 'MarkerSize', 5);
   title(‘Research of the “Galaxy”’),  hold on;  pause(.02)
end
```

The following cases were investigated with such a MATLAB-code [2] (1) for $z_0 = 0$, $C = -.38 - .587i$ (2) $z_0 = 0$, $C =$ and (3) $z_0 = 0$, $C =$. It was determined, and will be displayed in the presentation that they realize a strict periodicity, correspondingly: (1) 5-beam "Galaxy", (2) -beam, and (3) –beam "Galaxy".

So, a certain order and laws may exist even in the "chaos".

Supervisor: Prof. Ye.A.Gayev

References

1. Гринченко В.Т. и др. Фракталы: от удивления к рабочему инструменту: уч.пособие. К.: Наук. Думка, 2013. – 270 с.

2. Gayev Ye.A., Nesterenko B.N. MATLAB forMath and Programming. 2006.